Answer Key

PC11 Final Exam TERM 3 Review

Chapter 3&4: Quadratic Functions

1. Solve the following equations:

a) $x^2 - 2x - 7 = 0$

$x = \frac{2 \pm \sqrt{(-2)^2 - 4(-7)}}{2}$

$= \frac{2 \pm \sqrt{32}}{2}$

$= \frac{2 \pm 4\sqrt{2}}{2}$

$= 1 \pm 2\sqrt{2}$

$\therefore x_1 = 1 + 2\sqrt{2}$

$x_2 = 1 - 2\sqrt{2}$

b) $x^2 + 2x + 7 = 0$

$x = \frac{-2 \pm \sqrt{(2)^2 - 4(7)}}{2}$

$= \frac{-2 \pm \sqrt{-24}}{2}$ ← Not possible.

$\therefore$ No real solutions

c) $(2x+1)(x-1) = 5x$

$2x^2 - x - 1 = 5x$

$2x^2 - 6x - 1 = 0$

$x = \frac{6 \pm \sqrt{(-6)^2 - 4(2)(-1)}}{2(2)}$

$= \frac{6 \pm \sqrt{44}}{4}$

$= \frac{6 \pm 2\sqrt{11}}{4} = \frac{3 \pm \sqrt{11}}{2}$

$\therefore$ Solutions $x_1 = \frac{3 + \sqrt{11}}{2}$

$x_2 = \frac{3 - \sqrt{11}}{2}$

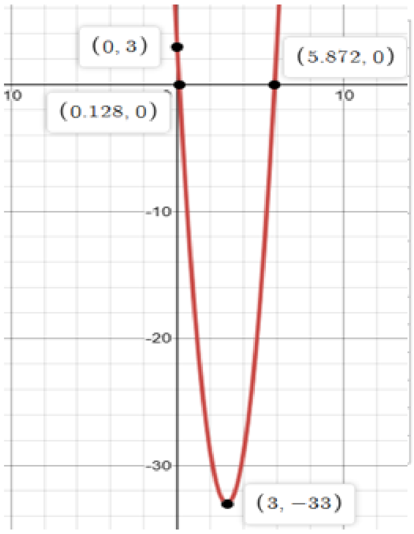
2. What is a perfect square trinomial?

A trinomial that can be factored as the square of a binomial.

$a^2 + 2ab + b^2 = (a+b)^2$

3. Graph the following equation $y = 4x^2 - 24x + 3$. Identify the vertex, intercepts, equation of a.o.s, domain and range

a.o.s: $x = -\frac{b}{2a} = \frac{24}{8} = 3$	X-Int: Use: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x_1 = \frac{6 + \sqrt{33}}{2}$ $x_2 = \frac{6 - \sqrt{33}}{2}$
vertex: $(3, -33)$	
Y-Int: $(0, 3)$	
Domain: $x \in R$	
Range: $y \geq -33, y \in R$	



4 Sketch a graph for y = -3x² – 4x + 4 . What is the axis of symmetry, vertex,x- intercept, y intercept,

4. Sketch a graph for y = -3x² – 4x + 4 . What is the axis of symmetry, vertex, x- intercept, y intercept, domain and range.

a.o.s: $x = -\frac{b}{2a} = \frac{4}{-6} = -\frac{2}{3}$

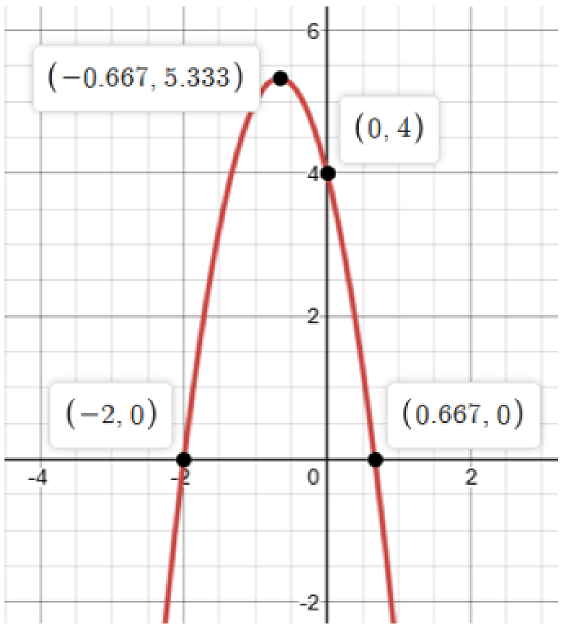
vertex: $\left(-\frac{2}{3}, \frac{16}{3}\right)$

X-Int: $(-2, 0)$ $\left(\frac{2}{3}, 0\right)$

vertex: $(0, 4)$

Domain: $x \in R$

Range: $y \leq \frac{16}{3}, y \in R$



5. For the quadratic equation $y = 3x^2+12x-2$

a. Without solving predict the zeros of this equation. (use discriminat)

$b^2-4ac = 12^2-4(3)(-2) = \underline{168}$ $\therefore$ 2 real roots.

b. Find the x and y intercepts. What do is the significance of these intercepts?

$\boxed{\text{y-int: } (0,-2)}$

$x = \frac{-12 \pm \sqrt{12^2-4(3)(-2)}}{6} = \frac{-12 \pm \sqrt{168}}{6}$

$= -\frac{6 \pm \sqrt{42}}{3}$ $\boxed{x_1 = 0.16 \quad x_2 = -4.16}$

$\boxed{\text{x-ints: } \left(\frac{-6+\sqrt{42}}{3}, 0\right) \text{ and } \left(\frac{-6-\sqrt{42}}{3}, 0\right)}$

c. Change the equation to standard form

$y = 3(x^2+4x)-2$

$y = 3(x^2+4x+4)-2-12$

$\boxed{y = 3(x+2)^2-14}$

d. What are the coordinates of the vertex?

$(-2,-14)$

e. What is the equation of axis of symmetry?

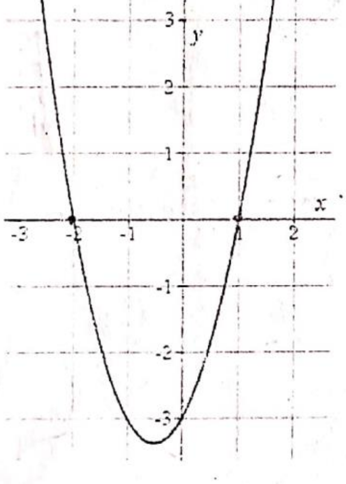
$x = -2$

f. What is the domain and range?

Domain: $x \in \mathbb{R}$

Range: $y \geq -14, y \in \mathbb{R}$

6. Write the equation of the following graph



Step 1: **Use Factored Form**

$$y = a(x-x_1)(x-x_2)$$
$$y = a(x+2)(x-1)$$

Step 2: **Use Point (0, -3) to find "a"**

$$-3 = a(0+2)(0-1)$$
$$-3 = -2a$$
$$a = \frac{3}{2}$$

Equation: $y = \frac{3}{2}(x+2)(x-1)$

7. What is the discriminate for $y = 4x^2 - 24x + 3$?

$b^2-4ac = (-24)^2-4(4)(3) = \underline{528}$

8. What does the discriminate need to be for a quadratic function to have no zeroes?

The discriminant must be negative in order to have no zeros. ($b^2-4ac < 0$.).

9. Given the zeroes of a quadratic function, 4 and -8. What is the equation of axis of symmetry?

$\frac{-8+4}{2} = \frac{-4}{2} = -2$ The equation of the axis of symmetry is $\underline{x = -2}$.

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10. Every week, a restaurant sells approximately 300 pizzas for \$2.50 each. Through market research, the restaurant manager determines that for every \$0.10 increase in price, they will sell 20 less pizzas. What is the price of a pizza that will maximize the revenue and what is the maximum revenue?

Let x be the amount of increase.

Idea: 300×2.5, 2.6×280, $2.7 \times 260 \ldots$

Revenue: $(300-20x)(2.5+0.1x)$.

$f(x) = 750 + 30x - 50x - 2x^2$

$f(x) = -2x^2 - 20x + 750$

$= -2(x^2 + 10x) + 750$

$= -2(x^2 + 10 + 25) + 750 + 50$

x is the level of increase it must be

$f(x) = -2(x+5)^2 + 800$

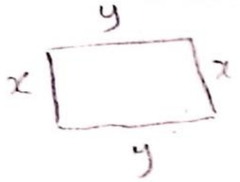
$\therefore$ vertex $(-5, 800)$

$Price = 2.50 + 0.1(-5)$

$Price = \$2.00$

$\therefore$ *The price of pizza should be* \$2.00 *to maximize the revenue of* \$800

11. A rectangular play area is to be bounded by 120 m of fencing. Determine the maximum area and the dimensions of this rectangle.



$2x + 2y = 120$

$x \cdot y = $ max area

$\begin{cases} y = 60 - x \\ x \cdot y \end{cases}$

$x(60-x) = -x^2 + 60x$

$f(x) = -(x^2 - 60x)$

$f(x) = -(x^2 - 2 \cdot 30x + 30^2 - 30^2)$

$f(x) = -(x-30)^2 + 900$

vertex $(30, 900)$

$xy = x(60-x)$

$xy = 30(60-30)$

$= 30 \cdot 30$

$= 900 m^2$

$\therefore$ The max area is $900 m^2$ with the dimensions $30m \times 30m$.

12. Every week, a take-out restaurant sells approximately 2000 chicken wraps for \$1.50 each. Through market research, the restaurant manager determines that for every \$0.10 increase in price, she will sell 100 fewer wraps. What is the price of a wrap that will maximize the revenue and what is the maximum revenue?

Let x be the amount of increase.

ex 2000×1.5 ; 1.6×1900 ; 1.7×1800

Revenue: $(2000 - 100x)(1.5 + 0.1x)$

$f(x) = 3000 + 200x - 150x - 10x^2$

$f(x) = -10x^2 + 50x + 3000$

$= -10(x^2 - 5x) + 3000$

$= -10(x - \frac{5}{2})^2 + 10 \cdot \frac{25}{4} + 3000$

$f(x) = -10(x - \frac{5}{2})^2 + 3062.5$

x is level of increase it must be

$\therefore$ vertex $(\frac{5}{2}, 3062.5)$

The price of a wrap should be $(1.5 + 0.1x = 1.5 + 0.25 = 1.75)$ \$1.75. The maximum revenue would be \$3062.50.

13. The graph of a quadratic function passes through A(-5, 8), and the x-intercepts of the function are -2 and 9. What is the equation of the graph in general form?

$y = a(x+2)(x-9)$

$8 = a(-5+2)(-5-9)$

$\frac{8}{42} = \frac{42a}{42}$

$a = \frac{4}{21}$

$y = \frac{4}{21}(x+2)(x-9)$

$y = \frac{4}{21}(x^2 - 7x - 18)$

$\boxed{y = \frac{4}{21}x^2 - \frac{4}{3}x - \frac{24}{7}}$

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14. The graph of a quadratic function passes through points A(2,5) and B(-4,-1). The axis of symmetry is x=1. What is the equation of the graph in standard form?

$y = a(x-p)^2 + q$

A.o.s $\Rightarrow x=1$: $y = a(x-1)^2 + q$

$\boxed{\therefore y = -\frac{1}{4}(x-1)^2 + \frac{21}{4}}$

Point A: $5 = a(2-1)^2 + q$

$5 = a + q$ ①

$q = 5 - a$

Point B: $-1 = a(-4-1)^2 + q$

$-1 = 25a + q$ ②

Put ① into ②: $-1 = 25a + 5 - a$

$-6 = 24a$

$\boxed{a = -\frac{1}{4}}$

$5 = -\frac{1}{4} + q$

$\boxed{q = \frac{21}{4}}$

15. The graph of a quadratic function passes through B(6, -60), and the zeros of the function are -4 and 2. Write the equation of the graph in general form.

$y = a(x+4)(x-2)$

$-60 = a(6+4)(6-2)$

$\frac{-60}{40} = \frac{40a}{40}$

$a = -\frac{3}{2}$

$y = -\frac{3}{2}(x+4)(x-2)$

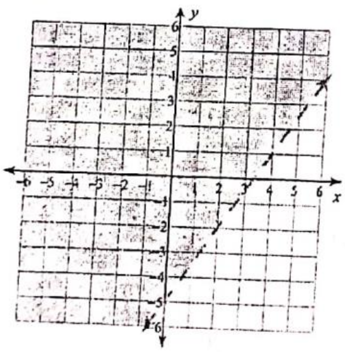
$y = -\frac{3}{2}(x^2 + 2x - 8)$

$\boxed{y = -\frac{3}{2}x^2 - 3x + 12}$

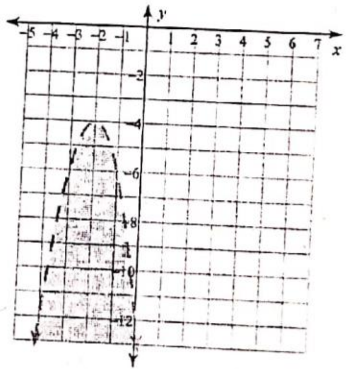
Chapter 5: Inequalities

1. Solve the quadratic inequality
   a. $0 > 2x^2 + 7x + 6$ $\quad -2 < x < -\frac{3}{2}$
   b) $0 \leq 12x^2 - 44x + 7$ $\quad x \leq \frac{1}{6}$ or $x \geq \frac{7}{2}$

2. Write an inequality to describe this graph



$y > \frac{3}{2}x - 5$



Step 1: Use standard Form

$$y = a(x-p)^2 + q$$
$$y = a(x+2)^2 - 4$$

Step 2: Use Point any point such as (-1, -6) to find "a"

$$a = -2$$

Step 3: Use a Test Point to find $<$ *or* $>$

Note: Because it is dashed line we know it cannot be $\leq$ *or* $\geq$

Equation: $y < -2(x+2)^2 - 4$

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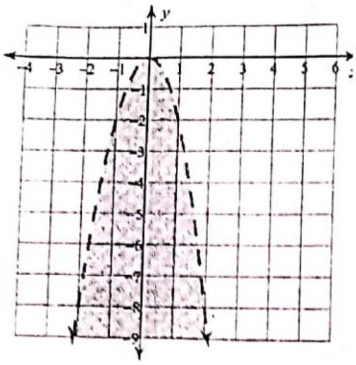
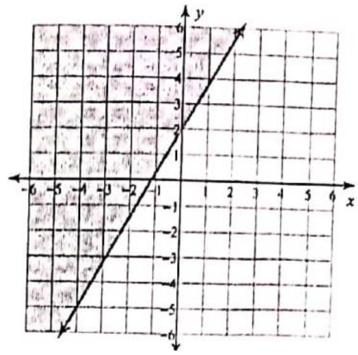
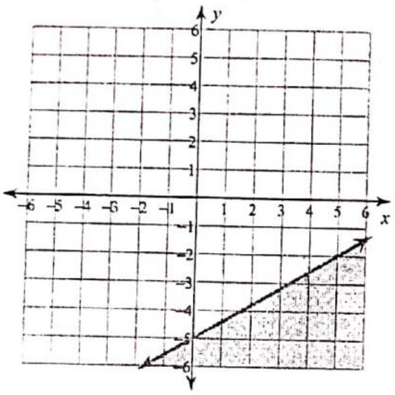


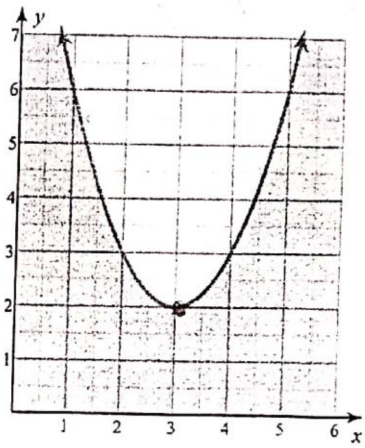
$y < -2x^2$



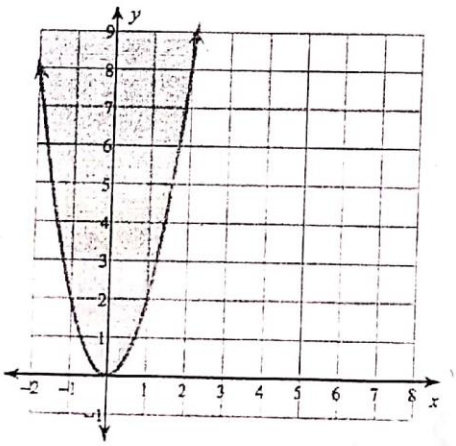
$$y \geq \frac{7}{4}x + 2$$



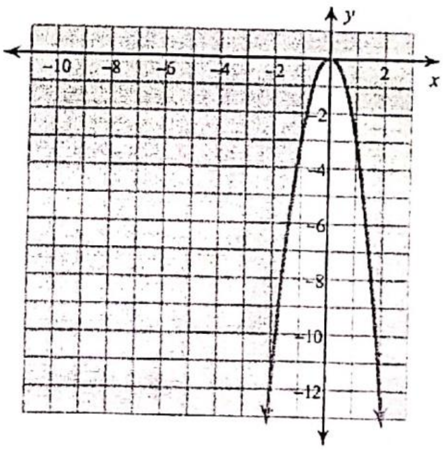
$y \leq \frac{3}{5}x - 5$



$y \leq (x-3)^2 + 2$



$y \geq 2x^2$



$y \geq -3x^2$

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3. Solve the systems

Check: Both $6 = 2(0)^2 + 7(0) + 6$, $6 = 6$ ✓ | $1 = 2(-1)^2 + 7(-1) + 6$, $1 = 2 + 6 - 7$, $1 = 1$ ✓

$y = 2x^2 + 7x + 6$ ①
$y - 5x = 6 \rightarrow y = 5x + 6$ ②

Put ② into ①

$5x + 6 = 2x^2 + 7x + 6$
$0 = 2x^2 + 2x$
$0 = 2x(x+1)$
$x_1 = 0$ or $x_2 = -1$

x_1: $y_1 - 5(0) = 6$
$\boxed{y_1 = 6}$

$\therefore \begin{cases} x_1 = 0 \\ y_1 = 6 \end{cases}$

x_2: $y_2 - 5(-1) = 6$
$y_2 = 6 - 5$
$\boxed{y_2 = 1}$

$\therefore \begin{cases} x_2 = -1 \\ y_2 = 1 \end{cases}$

① $y = -(x-3)^2 + 10$
② $y = 3x - 2$

Put ② into ①.

$3x - 2 = -(x-3)^2 + 10$
$3x - 2 = -x^2 + 6x - 9 + 10$
$x^2 - 3x - 3 = 0$
$x = \dfrac{3 \pm \sqrt{(-3)^2 - 4(-3)}}{2(1)} = \dfrac{3 \pm \sqrt{21}}{2}$
$x_1 = \dfrac{3 + \sqrt{21}}{2}$ $x_2 = \dfrac{3 - \sqrt{21}}{2}$

x_2: $y_2 = \dfrac{9 - 3\sqrt{21}}{2} - 2$
$y_2 = \dfrac{5 - 3\sqrt{21}}{2}$

x_1: $y_1 = \dfrac{9 + 3\sqrt{21}}{2} - 2$
$y_1 = \dfrac{5 + 3\sqrt{21}}{2}$

$\therefore$ Solutions
$\begin{cases} x_1 = \dfrac{3 + \sqrt{21}}{2} \\ y_1 = \dfrac{5 + 3\sqrt{21}}{2} \end{cases}$

$\begin{cases} x_2 = \dfrac{3 - \sqrt{21}}{2} \\ y_2 = \dfrac{5 - 3\sqrt{21}}{2} \end{cases}$

Check!

Chapter 7: Rational Expressions

1. Simplify the rational expression. State all the NON PERMISSIBLE values.

a. $\dfrac{9x+18}{x^2-2x-8} \cdot \dfrac{3x-12}{6x} = \dfrac{9\cancel{(x+2)}}{\cancel{(x-4)}\cancel{(x+2)}} \cdot \dfrac{3\cancel{(x-4)}}{6x}$

$= \dfrac{18}{6x} = 3x$ $x \neq 4, -2, 0$

b. $\dfrac{5n+15}{4n+8} \cdot \dfrac{2n+4}{3n+9} = \dfrac{5\cancel{(n+3)}}{{}_2\cancel{4}\cancel{(n+2)}} \cdot \dfrac{\cancel{2}\cancel{(n+2)}}{3\cancel{(n+3)}} = \dfrac{5}{6}$ $x \neq -2, -3$

c. $\dfrac{m^2-2m-8}{8m+24} \div \dfrac{2m-8}{m^2+7m+12}$

$= \dfrac{(m-4)(m+2)}{8(m+3)} \div \dfrac{2(m-4)}{(m+4)(m+3)}$

$= \dfrac{(m-4)(m+2)}{8\cancel{(m+3)}} \times \dfrac{(m+4)\cancel{(m+3)}}{2\cancel{(m-4)}}$

$= \dfrac{(m+2)(m+4)}{16}$, $m \neq -3, -4, 4$

d. $\dfrac{7x+4}{x^2+3x+2} - \dfrac{3x-2}{x^2+3x+2}$

$= \dfrac{7x + 4 - 3x + 2}{(x+2)(x+1)}$

$= \dfrac{4x+6}{(x+2)(x+1)} = \dfrac{2(2x+3)}{(x+2)(x+1)}$ $x \neq -2, -1$

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e. $\frac{1}{7(x-3)}+\frac{4}{7}=\frac{3}{(x-3)}$ $\rightarrow$ $\frac{1}{7(x-3)}+\frac{4x-12}{7(x-3)}=\frac{21}{7(x-3)}$

$\frac{4x-11}{7(x-3)}=\frac{21}{7(x-3)}$

$4x-11=21$

$\frac{4x}{4}=\frac{32}{4}$

$1x=8$

f. $\frac{3}{y+3}+\frac{2y}{y^2+7y+12}$

$=\frac{3}{y+3}+\frac{2y}{(y+4)(y+3)}=\frac{3(y+4)+2y}{(y+4)(y+3)}=\frac{5y+12}{(y+4)(y+3)}$ $\quad y\neq -4,-3.$

g. $\frac{1}{y+3}+\frac{4}{y^2+4y+3}$

$=\frac{1}{y+3}+\frac{4}{(y+3)(y+1)}=\frac{y+1+4}{(y+3)(y+1)}=\frac{y+5}{(y+3)(y+1)}$ $\quad y\neq -3,-1$

h. $\frac{2x+3}{5x-30}-\frac{3x+4}{x-6}$

$=\frac{2x+3}{5(x-6)}-\frac{3x+4}{x-6}=\frac{2x+3-15x-20}{5(x-6)}=\frac{-13x-17}{5(x-6)}$ $\quad x\neq 6$

i. $\frac{2}{5}-\frac{7}{(x+6)}=\frac{9}{5(x+6)}$

$\frac{2(x+6)}{5(x+6)}-\frac{7(5)}{5(x+6)}=\frac{9}{5(x+6)}$

$2(x+6)-7(5)=9$

$2x+12-35=9$

$2x-23=9$

$2x=32$

$\boxed{\boldsymbol{x=16}}$ $\quad x\neq -6$

j. $\frac{18}{5x+10}+\frac{4}{5}=\frac{-6}{x+2}$

$\frac{18}{5(x+2)}+\frac{4x+8}{5(x+2)}=\frac{-30}{5(x+2)}$.

$18+4x+8=-30$

$\frac{4x}{4}=\frac{-56}{4}$

$\boxed{x=-14}$ $\quad x\neq -2$

k. $\frac{2}{x-6}+\frac{7}{x+2}=\frac{4x+2}{x^2-4x-12}$

$\frac{2}{x-6}+\frac{7}{x+2}=\frac{4x+2}{(x-6)(x+2)}$

$\frac{2x+4+7x-42}{(x-6)(x+2)}=\frac{4x+2}{(x-6)(x+2)}$

$2x+4+7x-42=4x+2$

$2x+7x-4x=2+42-4$

$5x=40$

$\boxed{x=8}$

$x\neq 6$

$x\neq -2$

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2. 2.A boat travels at an average speed of 15 km/h in still water. The boat travels 12 km downstream in the same time as it travels 8 km upstream. Determine the average speed of the current.

$d = s \cdot t$

$t = \frac{d}{s}$

x = avg. speed of current

$$\frac{12}{15+x} = \frac{8}{15-x}$$

$$12(15-x) = 8(15+x).$$

$$180 - 12x = 120 + 8x$$

$$-20x = -60$$

$$x = 3.$$

$\therefore$ The average speed of the current is 3 km/h.

3. 3.A car travels from home to work at an average speed of 60 km/h, and because of traffic returns from work at an average speed of 40 km/h. What is the average speed for the entire trip?

Average Speed :

$$\frac{2}{x} = \frac{1}{60} + \frac{1}{40}.$$

$$240 = 2x + 3x$$

$$x = 48$$

$\therefore$ The average speed is 48 Km/h.

4. 4. How much lemon juice must be added to 2 L of water to make a lemonade solution that contains 20% lemon juice?

* Add x lemon juice.

$$\frac{x}{x+2} = \frac{20}{100}$$

$$100x = 20x + 40$$

$$80x = 40$$

$$x = \frac{1}{2}$$

$\therefore$ We should add $\frac{1}{2}$ L of lemon juice!

5. A plane flies 910 miles with the wind in the same time it can go 660 miles against the wind. The speed of the plane in still air is 305 miles per hour. What is the speed of the wind?

x = speed of the wind

$$\frac{910}{305+x} = \frac{660}{305-x}$$

$$910(305-x) = 660(305+x)$$

$$277550 - 910x = 201300 + 660x$$

$$\frac{76250}{1570} = \frac{1570x}{1570}$$

$$\frac{7625}{157} = x.$$

$$x \approx 48.567$$

$\therefore$ The speed of the wind is approximately 49 miles/hour.

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PC11 Final Exam TERM 4 Review

Chapter 2: Absolute Values and Radicals

1. Write each radical in simplest form. For what values of the variables is the radical defined?

a) $(\sqrt{3a^2b})(\sqrt{6ab^5})$

$(a\sqrt{3b})(b^2\sqrt{6ab})$

$= ab^2\sqrt{3\cdot 6\cdot a\cdot b\cdot b}$

$= ab^3\sqrt{18a}$

$= 3ab^3\sqrt{2a}$

$\{b \geq 0,\ b \in \mathbb{R}$; $a \in \mathbb{R}$.

b) $(4x\sqrt{10xy})(3y\sqrt{2x})$

$= 4x\cdot 3y\cdot\sqrt{10xy\cdot 2x}$

$= 4x\cdot 3y\cdot\sqrt{20x^2y}$

$= 24x^2y\sqrt{5y}$

$\{x \geq 0, \in \mathbb{R};\ y \geq 0, \in \mathbb{R}$

c) $(2x\sqrt[3]{2y^4})(x^2\sqrt[3]{4y^2})$

$= 2x^3\sqrt[3]{8y^6}$

$= 4x^3y^2$

$\{x, y \in \mathbb{R}$

d) $(ab\sqrt[3]{2ab^2})(3a\sqrt[3]{4a^2b^2})$

$= ab\cdot 3a\cdot\sqrt[3]{2ab^2\cdot 4a^2b^2}$

$= 3a^2b\cdot\sqrt[3]{8a^3b^4}$

$= 6a^3b^2\sqrt[3]{b}$

$\{a, b \in \mathbb{R}$

e) $\dfrac{9x^2\sqrt{x^2y^5}}{3x^5\sqrt{x^6y}}$

$= \dfrac{9x^3y^2\sqrt{y}}{3x^8\sqrt{y}}$

$= \dfrac{3y^2}{x^5}$

$\{y \geq 0,\ y \in \mathbb{R};\ x \in \mathbb{R}$

f) $\dfrac{\sqrt[3]{81x^2y^5}}{\sqrt[3]{x^5y}}$

$= \dfrac{3y\sqrt[3]{3x^2y}}{x\sqrt[3]{x^2y}}$

$= \dfrac{3y\sqrt[3]{3}\cdot\cancel{\sqrt[3]{x^2y}}}{x\cancel{\sqrt[3]{x^2y}}} = \dfrac{3y\sqrt[3]{3}}{x}$

$\{x, y \in \mathbb{R}$

2. Simplify each radical. For what values of the variables is the radical defined? State restrictions.

25) $-4\sqrt{216x^2y^2z}$

$= -4\cdot 6\cdot x\cdot y\sqrt{6z}$

$= -24xy\sqrt{6z}$

Defined for: $\{z \geq 0,\ x, y, z \in \mathbb{R}$

26) $-3\sqrt{24a^4b^2c^3}$

$= -3\cdot 2\cdot a^2\cdot b\cdot c\sqrt{6c}$

$= -6a^2bc\sqrt{6c}$

$\{c \geq 0,\ abc \in \mathbb{R}$

27) $3\sqrt{16x^4y^4z}$

$= 12xy\sqrt{z}$

$\{z \geq 0,\ x, y, z \in \mathbb{R}$

28) $-2\sqrt{48a^3b^4c^2}$

$= -2\cdot 4\cdot a\cdot b^2\cdot c\sqrt{3a}$

$= -8ab^2c\sqrt{3a}$

$\{a \geq 0,\ a, b, c \in \mathbb{R}$

29) $6\sqrt{75mp^2q^3}$

$= 6\cdot 5\cdot p\cdot q\sqrt{3mq}$

$= 30pq\sqrt{3mq}$

$\{m \geq 0,\ q \geq 0,\ m, p, q \in \mathbb{R}$

30) $4\sqrt{36x^2y^3z^4}$

$= 4\cdot 6\cdot x\cdot y\cdot z^2\sqrt{y}$

$= 24xyz^2\sqrt{y}$

Defined for $\{y \geq 0,\ x, y, z \in \mathbb{R}$

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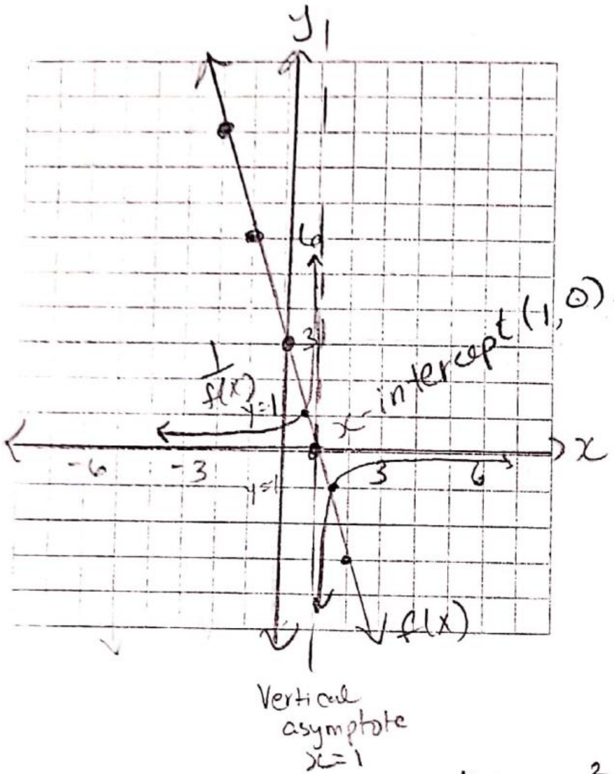
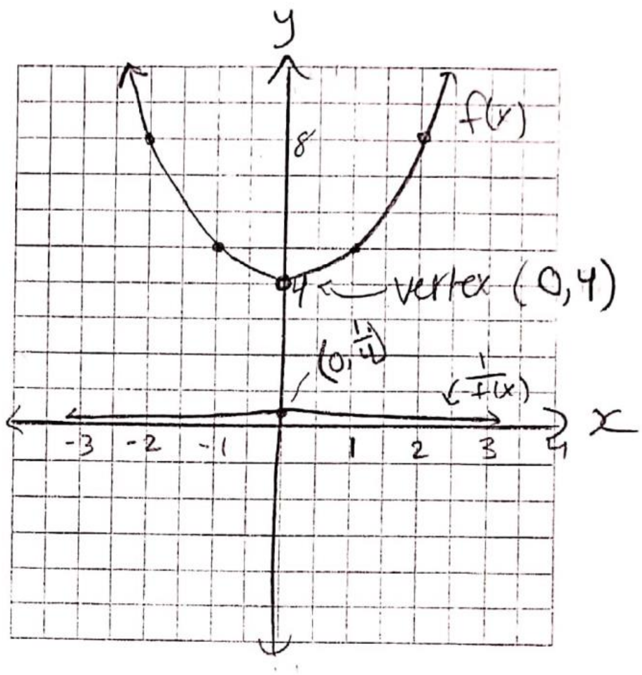
1. For each graph, drawn y= f(x) and reciprocal functions $y = \frac{1}{f(x)}$. Label all the important points. State the domain and range for both. Indicate the equation of the vertical and horizontal asymptotes.

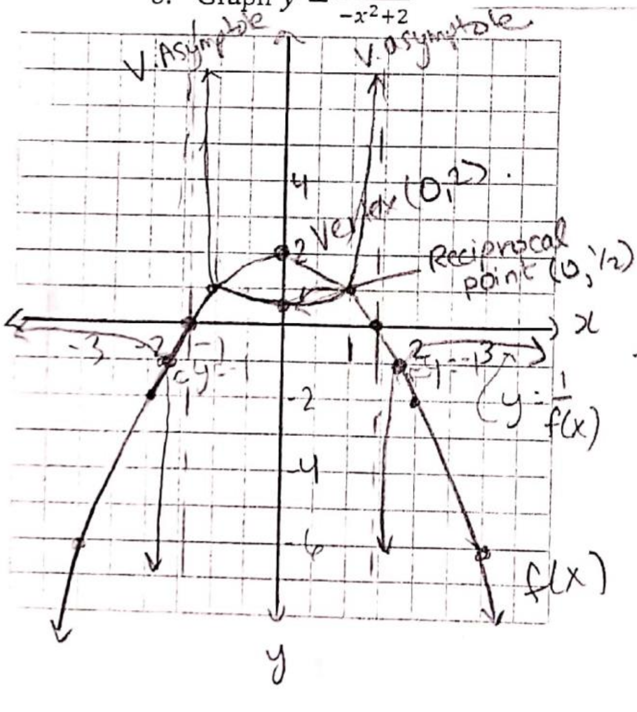
a. Graph $y = \frac{1}{-3(x-1)}$ $f(x) = -3x+3$



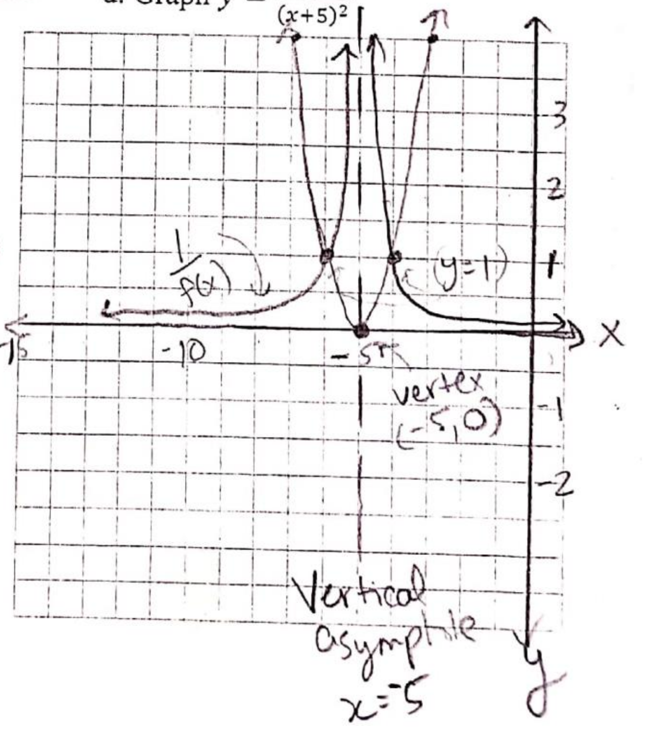
c. graph $y = \frac{1}{x^2+4}$ $f(x) = x^2+4$



b. Graph $y = \frac{1}{-x^2+2}$ $f(x) = -x^2+2$



d. Graph $y = \frac{1}{(x+5)^2}$ $f(x) = (x+5)^2$



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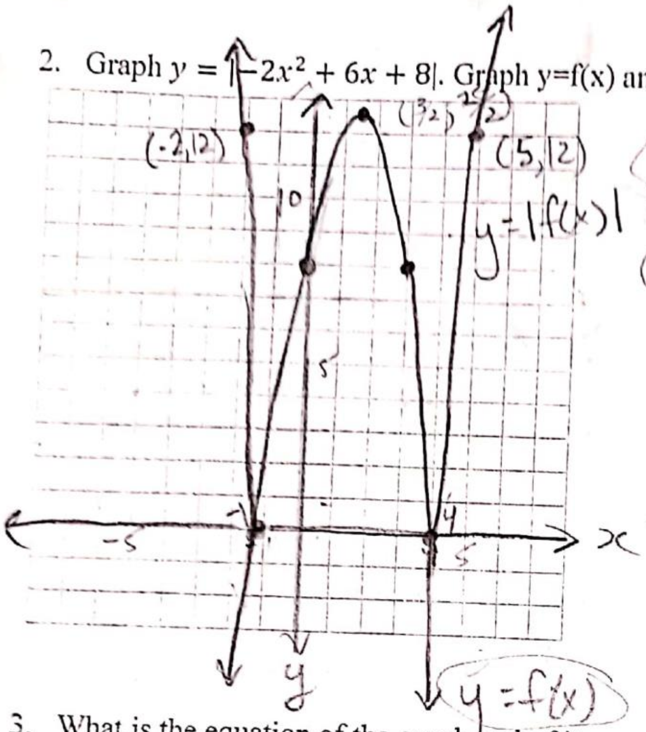
2. Graph $y = |-2x^2 + 6x + 8|$. Graph y=f(x) and y=|f(x)|. State the domain, range and the critical points.



$y = -2(x^2 - 3x - 4)$
$y = -2(x-4)(x+1)$

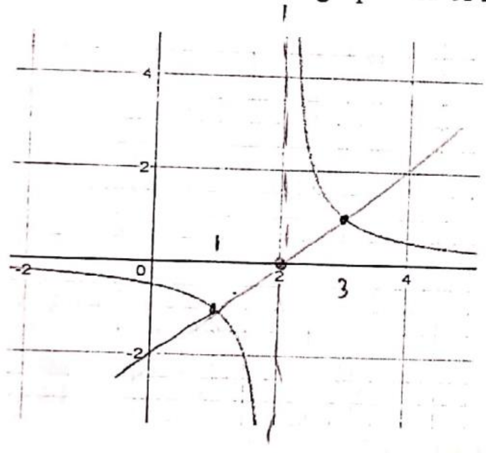
Critical points:
(-1,0) and (4,0)

Domain: $x \in \mathbb{R}$

Range for $y=f(x)$: $y \leq \frac{25}{2}, y \in \mathbb{R}$
Range for $y=|f(x)|$: $y \geq 0, y \in \mathbb{R}$

3. What is the equation of the graph and of its corresponding linear or quadratic equation?

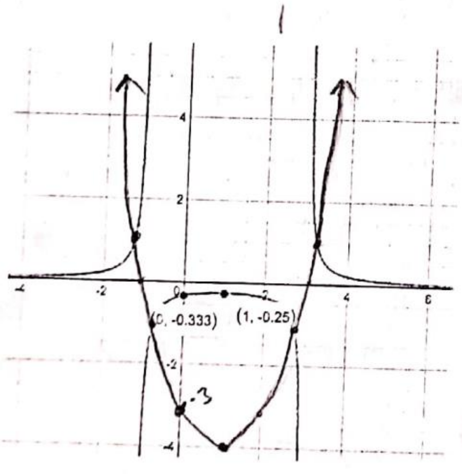
a.



Linear Equation:
$y = x - 2$

Reciprocal:
$y = \frac{1}{x-2}$

b.



Quadratic equation:

$y = x^2 - 2x - 3$.

Reciprocal graph:

$y = \frac{1}{x^2 - 2x + 3}$

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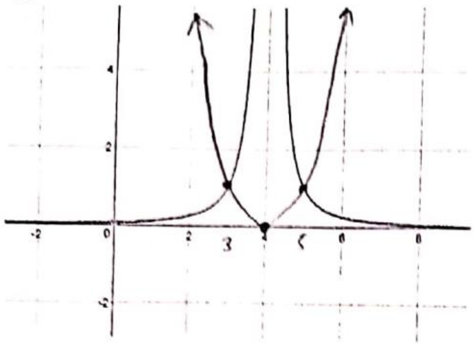
c.



Quadratic

$y=(x-4)^2$

or

$y=x^2-8x+16$

Reciprocal:

$y=\frac{1}{(x-4)^2}$

4. Solve algebraically. Remember to check solutions.

$\therefore$ Solutions are $x=12, -2, 6$ or 4.

a. $6x = |x^2+9|$

If $|x^2+9| \geq 0$

$6x = x^2+9$

$0=x^2-6x+9$

$0=(x-3)(x-3)$

$\therefore \boxed{x=3}$

If $|x^2+9|<0$

$6x=-x^2-9$

$x^2+6x+9=0$

$(x+3)(x+3)=0$

$\therefore \boxed{x=-3}$

Check

$6(3)=|(3)^2+9|$

$18=|9+9|$

$18=18$ ✓

$6(-3)=|(-3)^2+9|$

$-18=|9+9|$

$-18=18$ ✗

$\therefore$ $x=3$ is the only solution

b. $|2x-4| = 7+x$

$2x-4=7+x$

$\boxed{x=11}$

$-2x+4=7+x$

$-3x=3$

$\boxed{x=-1}$

$\therefore$ Solutions $x=11$ and -1

Check: $|2(11)-4|=7+11$

$|18|=18$

$18=18$ ✓

$|2(-1)-4|=7-1$

$|-6|=7-1$

$6=6$ ✓

c. $24 = |x^2-10x|$

$24=x^2-10x$

$0=x^2-10x-24$

$0=(x-12)(x+2)$

$x=12$ or $x=-2$

Check: $24=|(12)^2-10(12)|$

$24=|24|$

$24=24$ ✓

$24=|(-2)^2-10(-2)|$

$24=|24|$ ✓

$24=-x^2+10x$

$x^2-10x+24=0$

$(x-6)(x-4)=0$

$\boxed{x=6 \text{ or } x=4}$

$24=|(6)^2-10(6)|$

$24=|-24|$

$24=24$ ✓

$24=|(4)^2-10(4)|$

$24=|-24|$

$24=24$ ✓

d. $|5-3x|+12=31$

$5-3x+12=31$

$-3x=14$

$\boxed{x=-14/3}$

Check: $|5-3(\frac{-14}{3})|+12=31$

$|19|+12=31$

$31=31$ ✓

$-5+3x+12=31$

$3x=24$

$\boxed{x=8}$

$|5-3(8)|+12=31$

$|-19|+12=31$

$31=31$ ✓

5. a. If a graph has vertical asymptotes at x=6, what is a possible equation of the reciprocal function.

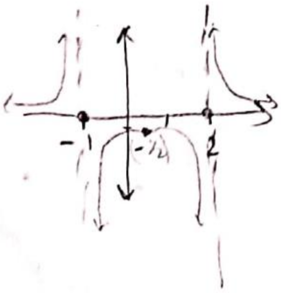
V. asymptote @ $x=6$

Reciprocal f(x) → x-int @ (6,0)

$\therefore$ Possible equation could be $y=(x-6)^2$

So reciprocal function would be $y=\frac{1}{(x-6)^2}$

b. If a graph has a vertical asymptote at x=2 and x=-1, what is a possible equation of the function.



$y=(x+1)(x-2)$

$y=x^2-x-2$

$f(x)=x^2-x-2$

$\therefore$ Reciprocal function could be $\frac{1}{(x+1)(x-2)}$

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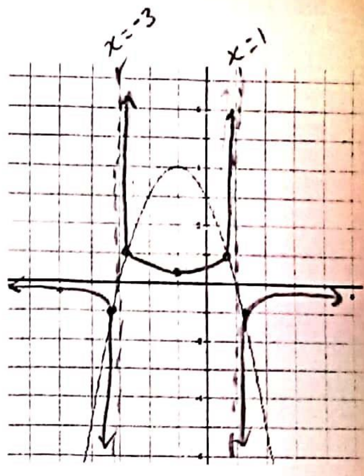
6. Sketch the graph of the corresponding reciprocal function



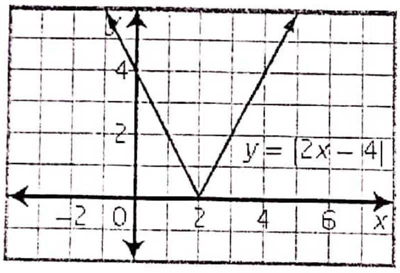
Vertex = (-1,4) ∴ Reciprocal point @ $(-1, \frac{1}{4})$

Graph touches same points at $y=1$ and $y=-1$.

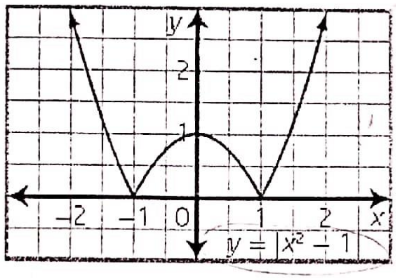
Horizontal asymptote @ $y=0$ and vertical asymptotes @ $x=-3$ and $x=1$.

7. What is a possible equation for each graph

a)



b)



$y = |2x-4|$

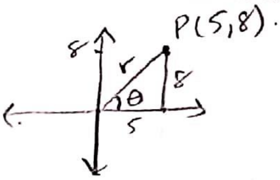
OR $y = |-2x+4|$

$y = |x^2-1|$

OR $y = |-x^2+1|$

Chapter 6: Trigonometry

1. The Point (5,8) is on the terminal arm of an angle θ in standard position.
   a. Determine the distance r from the origin to P.



$r^2 = 5^2 + 8^2$

$r^2 = 89$

$r = \sqrt{89}$

   b. Determine the primary trigonometric ratios of θ.

$\sin\theta = \frac{8}{\sqrt{89}} = \frac{8\sqrt{89}}{89}$

$\tan\theta = \frac{8}{5}$

$\cos\theta = \frac{5}{\sqrt{89}} = \frac{5\sqrt{89}}{89}$

   c. Determine the measure of θ to the nearest degree

$\theta = \tan^{-1}(8/5)$

$= 57.995$

∴ The measure of $\angle\theta$ is $58°$.

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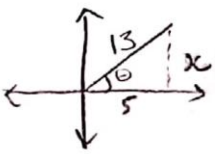
2. If $\cos\theta = \frac{5}{13}$, with θ in the first quadrant, determine $\sin\theta$ and $\tan\theta$. Determine the reference angle and the angle in standard position.

$\cos\theta = \frac{5}{13}$ (adj) / (hyp).



$x = \sqrt{13^2 - 5^2}$

$x = \sqrt{144}$

$\boxed{x = 12}$

So, $\sin\theta = \frac{12}{13}$

$\tan\theta = \frac{12}{5}$

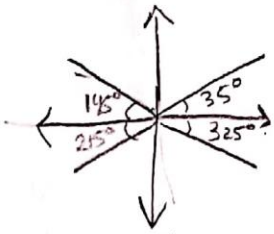
$\theta_R = \tan^{-1}(12/5)$

$= 67.38$

$\theta_R = \theta$ b/c it's in the first quadrant

$\therefore$ θ_R and θ are $67.4°$.

3. Determine all the angles between 0° to 360° in standard position that have a reference angle of 35 degrees. Draw all the angles in standard position.



Q2: $180° - 35° = 145°$

Q3: $180° + 35° = 215°$

Q4: $360° - 35° = 325°$

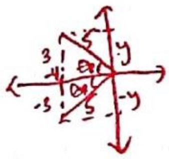
$\therefore$ The angles are 35°, 145°, 215° and 325°

4. Given that $\cos\theta = -\frac{4}{5}$, determine the other primary trig ratios of the angle. To the nearest degree, determine the possible values for θ when $0° \le \theta \le 360°$

* Cosine is negative so θ is in Quadrants 2 and 3.

Find θ_R: $\theta_R = \cos^{-1}(4/5)$ use Positive!

$\theta_R = 37°$



$y = \pm\sqrt{5^2 - (-4)^2}$

$y = \pm 3$

In Q2
$\sin\theta = \frac{3}{5}$
$\tan\theta = \frac{-3}{4}$

In Q3
$\sin\theta = -3/5$
$\tan\theta = 3/4$

These are the primary trig ratios. Make sure you give different quadrants!!

$\therefore$ Since θ is in Q2 or Q3:

$\theta_1 = 180 - 37 = 143°$.

$\theta_2 = 180 + 37 = 217°$.

θ is 143° or 217°

5. To the nearest degree, which values of θ satisfy the equation $\tan\theta = -\frac{4}{3}$ for $0° \le \theta \le 360°$?

$\theta_R = \tan^{-1}(4/3) \approx 53°$.

* tan is negative in Q2 + Q4.

Q2: $\theta = 180° - 53° = \boxed{127°}$

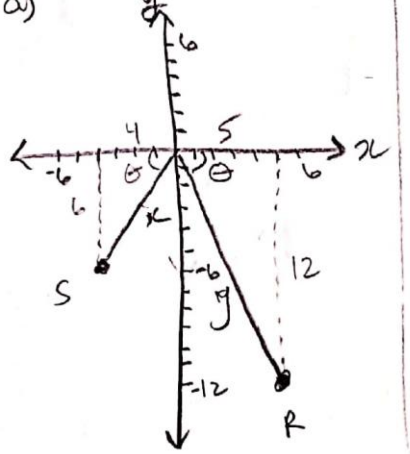
Q4: $\theta = 360 - 53° = \boxed{307°}$

6. Plot the following points on a coordinate grid.

R (5, -12) S (-4, -6)

a) Sketch each angle in standard position so that the terminal arm passes through each point
b) Determine the exact values of the sine, cosine and tangent ratios for each angle formed.
c) Calculate the measure of the angle in standard position.

a)



b) R(5, -12).

$y = \sqrt{5^2 + 12^2}$

$y = 13$

$\sin\theta = \frac{-12}{13}$

$\cos\theta = \frac{5}{13}$

$\tan = \frac{-12}{5}$

S(-4, -6)

$x = \sqrt{4^2 + 6^2}$

$x = 2\sqrt{13}$

$\sin\theta = \frac{-6}{2\sqrt{13}} = \frac{-3\sqrt{13}}{13}$

$\cos\theta = \frac{-4}{2\sqrt{13}} = \frac{-2\sqrt{13}}{13}$.

$\tan\theta = \frac{6}{4} = \frac{3}{2}$

For R:

$\theta_R = \tan^{-1}(12/5) = 67.4°$

$\therefore \theta = 360° - 67.4 = 292.6$

For S:

$\theta_R = \tan^{-1}(3/2) = 56.3°$

$\therefore \theta = 180° + 56.3° = 236.3°$

So the angles are 292.6° and 236.3°.

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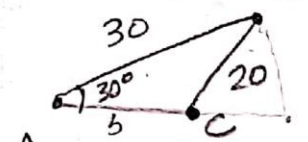
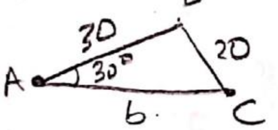
7. Give triangle ABC, with angle A, side AB and side BC. Complete the chart to summarize how to get each possible solution.

Description of possible triangles	Ratio
No Triangle	$\frac{BC}{AB} < \sin A$
1 Right Triangle	$\frac{BC}{AB} = \sin A$
1 Isosceles Triangle	$\frac{BC}{AB} = 1$
1 Scalene Triangle	$BC/AB > 1$
2 Scalene Triangles	$\sin A < BC/AB < 1$

8. One fire ranger station at A reports smoke 30 km away in a direction E30°N at B. A second station at C due east of the first station reports the smoke is 20 km away. To the nearest tenth of a kilometer, determine the distance between the two stations.

Ambiguous case!





Find "b"

Case 1:

$\frac{\sin 30}{20} = \frac{\sin C}{30}$

$\sin C = \frac{30\sin 30}{20}$

$= 0.75$

$\angle C = \sin^{-1}(0.75)$
$= 48.6$

$\angle B = 180° - 30 - 48.6$
$= 101.4°$

$\frac{\sin 101.4°}{b} = \frac{\sin 30°}{20}$

$b = \frac{20\sin 101.4°}{\sin 30°}$

$= 39.2$

Case 2:

$\angle C = 180° - 48.6$
$= 131.4°$

$\angle B = 180 - 131.4 - 30$
$= 18.6°$

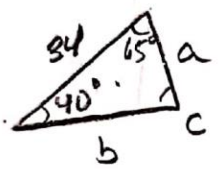
$\frac{\sin 18.6}{b} = \frac{\sin 30}{20}$

$b = \frac{20\sin(18.6)}{\sin 30}$

$= 12.76$

∴ The stations are either 39.2 km apart or 12.8 km apart.

9. The longest side of a triangle is 34'. The measures of two angles of the triangle are 40 and 65. Find the lengths of the other two sides.



$\angle C = 180 - 40 - 65 = 75°$.

* The longest side must be across from the biggest angle!

$\frac{\sin 75}{34} = \frac{\sin 40}{a}$

$a = \frac{34\sin(40)}{\sin 75}$

≈ 22.6

$\frac{\sin 75}{34} = \frac{\sin 65}{b}$

$b = \frac{34\sin(65)}{\sin 75}$

$= 31.9$

∴ The other two sides are 22.6' and 31.9'.

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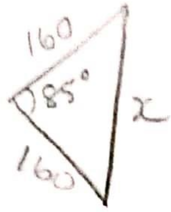
10. A house is built on a triangular plot of land. Two sides of the plot are 160 feet long and they meet at an angle of 85º. If a fence is to be built around the property, how much fencing is needed?



$x^2 = 160^2 + 160^2 - 2(160)(160) \cdot \cos(85)$

$x^2 \; 46737.626$

$x = 216.188$

Perimeter $= 160$ ft $+ 160$ ft $+ 216.188$ ft $= 536.188$ ft

∴ You will need about 537 ft of fencing to go around the property

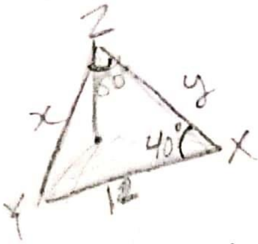
1. A post is supported by two wires (one on each side going in opposite directions) creating an angle of 80° between the wires. The ends of the wires are 12m apart on the ground with one wire forming an angle of 40° with the ground. Find the lengths of the wires.

2. Two ships are sailing from Halifax. The Nina is sailing due east and the Pinta is sailing 43° south of east. After an hour, the Nina has travelled 115km and the Pinta has travelled 98km. How far apart are the two ships?

3. 3 friends are camping in the woods, Bert, Ernie and Elmo. They each have their own tent and the tents are set up in a Triangle. Bert and Ernie are 10m apart. The angle formed at Bert is 30°. The angle formed at Elmo is 105°. How far apart are Ernie and Elmo?

(A) (B) (C)

11.



$\frac{\sin 80}{12} = \frac{\sin 40}{x}$

$x = \frac{12 \sin 40}{\sin 80} = 7.83$

$\angle y = 180 - 80 - 40 = 60°$

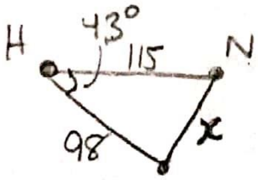
$\frac{\sin 60°}{y} = \frac{\sin 80}{12}$

$y = \frac{(\sin 60)(12)}{\sin 80}$

$y = 10.55$

∴ The wires are 7.8 m and 10.6 m long.

12.



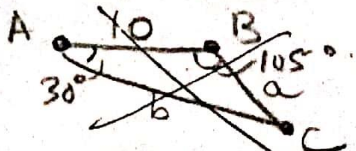
$x^2 = 98^2 + 115^2 - 2(98)(115) \cdot \cos(43)$

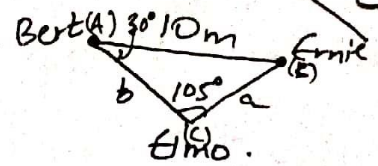
$x^2 = 6344.2876$

$x = 79.65$

∴ The two ships are approximately 80 km apart

13.





$\angle B = 180° - 105° - 30° = \underline{45°}$

$\frac{\sin 105}{10} = \frac{\sin 30}{a}$

$a = \frac{(\sin 30)(10)}{\sin 105} \approx 5.18$

∴ Ernie and Elmo are 5.18 m apart

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Chapter 1: Sequence and Series

1. Identify this series as arithmetic or geometric, then determine its sum.

$4 + 2.5 + 1 + \ldots. -32$

Arithmetic or Geometric: Arithmetic

$d = -1.5$

Find n:

$-32 = 4 + (n-1)(-1.5)$

$-36 = -1.5n + 1.5$

$-37.5 = -1.5n$

$\boxed{n = 25}$

$S_n = \frac{n(t_1 + t_n)}{2} = \frac{25(4-32)}{2}$

$= -350.$

∴ The sum of the series is -350.

2. One of Van Gogh's painting was appraised at \$250000. The value of the carving is estimated to increase by 12% each year. What will be the approximate value of the painting after 15 years?

$t_1 = 250{,}000$

$r = 12\%$ increase $= 1 + 0.12 = 1.12$

$n = 15$

$t_{15} = ?$

$t_{15} = t_1 r^{15-1}$

$t_{15} = 250{,}000\,(1.12)^{14}$

$t_{15} = 1221778.071$

∴ The approximate value of the painting in 15 years is \$1,221,778.

3. Find the sum of the first 12 terms for the series $8 + 2 + (-4) + (-10) + \ldots$

$d = -6$

$t_1 = 8$

$n = 12$

$S_n = \frac{n(2t_1 + d(n-1))}{2}$

$= \frac{12(2(8) - 6(12-1))}{2} = -300$

∴ The sum of the first 12 terms is -300.

4. Find the sum of the first 76 terms for the series $6 + 14 + 22 + 30 + \ldots$

$d = 8$

$t_1 = 6$

$n = 76$

$S_n = \frac{76(2(6) + 8(76-1))}{2}$

$= 23256$

∴ The sum of the first 76 terms is 23256.

5. An infinite geometric series with $r = -\frac{1}{9}$ is represented by this equation: $t_n = -5\left(-\frac{1}{9}\right)^{n-1}$

a. Determine the first 4 terms of the series

$t_1 = -5(-1/9)^{1-1} = -5$

$t_2 = -5(-1/9)^1 = 5/9$

$t_3 = \frac{-5}{81}$

$t_4 = 5/729$

∴ The first 4 terms are $-5, 5/9, -5/81, 5/729$.

b. Determine whether the series diverges or converges

The series converges, $r = -1/9$ ∴ $-1 < r < 1$.

c. If the series has a finite sum, determine the sum.

$S_\infty = \frac{t_1}{1-r} = \frac{-5}{1 + 1/9} = \cancel{-5} \cdot \frac{9}{\cancel{10}_2} = \frac{-9}{2}$

∴ The finite sum is $-9/2$.

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6. You invest \$120 000 in the bank on your 50th birthday. Every year the interest in your account grows 5% per year. How much money will have accumulated in your account on your 65th birthday?

$t_1 = 120000$

$r = 1 + 0.05 = 1.05$

$n = 65 - 50 = 15$

$t_{15} = t_1 \cdot r^{15-1}$

$t_{15} = 120{,}000\,(1.05)^{15}$

$= 237591.79$ → change

∴ You will have about \$237,591.79 in your account.

7. The circumference of a ripple made by a rock is 4cm, this circumference increases by 7% indefinitely. What is the sum of the circumference of all the ripples?

infinite.

$r = 1 + 0.07 = 1.07 \rightarrow (r > 1)$

$t_1 = 4$ cm

ELIMINATE THIS Question!

8. Identify each infinite geometric series that converges. Determine the sum of any series that converges

a. 2-3+4.5-6.75+…

$S_1 = 2$

$S_2 = -1$

$S_3 = 3.5$

$S_4 = -3.25$

∴ This series diverges.

b. 1/3 +2/9 +4/27 +8/81 +…..$\frac{16}{243} + \frac{32}{729} + \ldots \frac{64}{2187}$

$r = \frac{2}{3}$

$S_1 = \frac{1}{3} \approx 0.333$

$S_2 = \frac{5}{9} \approx 0.556$

$S_3 = \frac{19}{27} \approx 0.704$

$S_4 = \frac{65}{81} \approx 0.803$

$S_5 = \frac{211}{243} \approx 0.868$

$S_6 = \frac{665}{729} \approx 0.912$

$S_\infty = \frac{\frac{1}{3}}{\frac{1}{3}} = 1$

∴ Convergent to 1

9. An arithmetic series has t_1=5.5 and d=-2.5. Determine S_{40}.

$t_1 = 5.5$

$d = -2.5$

$n = 40$

$S_{40} = \frac{40(2(5.5) - 2.5(40-1))}{2}$

$= -1730$

∴ S_{40} is -1730.

10. Find the first 3 terms of the arithmetic series with $t_1 = 14$, $S_n = -1207$, $t_n = -85$

$S_n = \frac{n(t_1 + t_n)}{2}$

$-1207 = \frac{n(14-85)}{2}$

$-2414 = -71n$

$\boxed{n = 34}$

$t_n = t_1 + (n-1)d$

$-85 = 14 + (34-1)d$

$\frac{-99}{33} = \frac{33d}{33}$

$\boxed{-3 = d}$

$t_1 = 14$

$t_2 = 14 - 3 = 11$

$t_3 = 11 - 3 = 8$

∴ First 3 terms are 14, 11 and 8.

11. Determine the first term of the geometric sequence $t_8 = \frac{1}{4}$ and $r = \frac{1}{4}$

$t_n = t_1 r^{n-1}$

$\frac{1}{4} = t_1 \left(\frac{1}{4}\right)^{8-1}$

$\frac{1}{4} = t_1 \cdot \frac{1}{16384}$

$t_1 = 4096$

∴ The first term of the sequence is 4096.

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